7

INTRAMATHS OLYMPIAD

CLASS - XI

1. $7\sin x + 3\cos x = 7\cos x + 3\sin x$

$4\sin x = 4\cos x$

$\tan x = 1$

$x = \pi/4$

$y = \frac{7}{\sqrt{2}} + \frac{3}{\sqrt{2}} = \frac{10}{\sqrt{2}} = \boxed{5\sqrt{2}}$ $\boxed{B}$

2. Let $\sqrt{2-x} = t \Rightarrow 2-x = t^2 \Rightarrow 2-t^2 = x$

$\therefore 2-t^2 < \sqrt{t^2}$

$2-t^2 < t$

$t^2+t-2 > 0$

$(t-1)(t+2) > 0$

$-\infty \quad -2 \quad 1 \quad \infty$

$t \in (-\infty,-2) \cup (1,\infty)$

$t^2 \in (1,\infty)$

$1 < 2-x < \infty$

$-1 > x-2 > -\infty$

$1 > x > -\infty$

$\boxed{x \in (-\infty, 1)}$ $\boxed{A}$

3. $10 \leq 4n+1 \leq 99$

$\frac{9}{4} \leq n \leq \frac{98}{4}$

$2.25 \leq n \leq 24.5$

$3, 4, 5, \ldots 24$

$\therefore \; 24 - 3 + 1 = \boxed{22}$ $\boxed{C}$

4. $3\sin^2 x - 7\sin x + 2 = 0$

$(3\sin x - 1)(\sin x - 2) = 0$

$\sin x = \frac{1}{3}$

$\therefore$ in the interval $[0, 5\pi]$ the values of x is $\boxed{6}$ $\boxed{C}$

5. $2\left[\cos^2\frac{\pi}{8} + \cos^2\frac{3\pi}{8}\right]$

$= 2\left[\frac{1+\cos\pi/4}{2} + \frac{1+\cos 3\pi/4}{2}\right]$

$= \left[2 + \frac{1}{\sqrt{2}} + \cos\left(\pi - \frac{\pi}{4}\right)\right]$

$= \left[2 + \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right] = \boxed{2}$ $\boxed{A}$

6. $\sin\theta + \cos\theta = \frac{5}{13} - \frac{12}{13} = \boxed{\frac{-7}{13}}$ $\boxed{A}$

7. $x - 2 = \sqrt{3}i$

$x^2 + 4 - 4x = -3$

$x^2 - 4x + 7 = 0$

$4x+7$

$$
\begin{array}{r|l}
x^2-4x+7 & x^4 - 4x^2 + 8x + 35 \quad (x^2 + 4x + 5 \\
 & \cancel{x^4} - 4x^3 + 7x^2 \\
 & - \quad + \quad - \\
\hline
 & \cancel{4x^3} - 11x^2 + 8x + 35 \\
 & \cancel{4x^3} - 16x^2 + 28x \\
 & - \quad + \quad - \\
\hline
 & 5x^2 - 20x + 35 \\
 & 5x^2 - 20 + 35 \\
\hline
 & 0 \longrightarrow \boxed{D}
\end{array}
$$

8 EHMORT

E _ _ _ _ _ 5! = 120

H _ _ _ _ _ 5! = 120

M E _ _ _ _ 4! = 24

M H _ _ _ _ 4! = 24

M O E _ _ _ 3! = 6

M O H _ _ _ 3! = 6

M O R _ _ _ 3! = 6

M O T E _ _ 2! = 2

M O T H E R 1

309 $\boxed{C}$

9. D

10 $[x]^2 + 3[x] + 2 = 0$

$([x]+2)([x]+1) = 0$

$[x] = +2 \quad , \quad [x] = +1$

Solution $[1, 3)$ $\boxed{D}$

11 (C)

12 C

13 $X = 8$

$Y = 2$

$Z = 5$

$X + Y + Z = 15$ $\boxed{B}$

14 D

15 102 (C)

16 b, a, c

$a = \frac{b+c}{2}$

b, G_1, G_2, c

$G_1 = br = b\left(\frac{c}{b}\right)^{\frac{1}{3}} = c^{1/3} b^{2/3}$

$G_2 = br^2 = c^{2/3} b^{1/3}$

$G_1^3 + G_2^3$

$(c^{1/3} b^{2/3})^3 + (c^{2/3} b^{1/3})^3$

$cb^2 + bc^2$

$bc(b+c)$

$bc(2a)$

$2abc$ (B)

17. Slope $RQ = \frac{4-0}{2-4} = -2$

eq. of line $y - 0 = -2(x-4) \Rightarrow y = -2x + 8$

$\therefore P(0, 8)$

$\therefore$ ar $(\Delta OPQ) = \frac{1}{2} \cdot 4 \cdot 8 = \underline{16}$. (E)

18. $(\cos 12^\circ + \cos 132^\circ) + (\cos 84^\circ + \cos 156^\circ)$

$= 2\cos 72 \cos 60 + 2 \cos 120 \cos 36$

$= 2\cos 72 (\frac{1}{2}) + 2(-\frac{1}{2}) \cos 36$

$= \cos 72 - \cos 36 = \cos(90-18) - \cos 36 = \sin 18 - \cos 36$

$= \frac{\sqrt{5}-1}{4} - \frac{\sqrt{5}+1}{4} = -\frac{1}{2}$ $\boxed{E}$

19. If sum should be divisible by 3

$\therefore 0+1+2+4+5 = 12$ divisible by 3

so the 5-digit no. using 0, 1, 2, 4 and 5. $\underline{4} \times \underline{4} \times \underline{3} \times \underline{2} \times \underline{1} = 96$

If we take the digits 1, 2, 3, 4, 5 then their sum

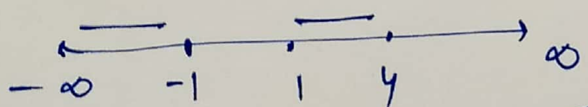
$1+2+3+4+5 = 15$ divisible by 3.

so the five digit no. $\underline{5}\ \underline{4}\ \underline{3}\ \underline{2}\ \underline{1} = 120$

$\therefore$ total no. of ways $120 + 96 = \underline{216}$ $\boxed{D}$

20. $4 - x \geq 0$, $x^2 - 1 > 0$

$x \leq 4$ $\quad -1 > x > 1$



$(-\infty, -1) \cup (1, 4]$ $\boxed{A}$

21 $\boxed{C}$ $ar(\Delta) = 8$.

22 $\boxed{B}$ $\frac{-144}{55}$.

23 $\boxed{A}$

24 $\boxed{C}$

25 $\boxed{D}$